

**Hale School**

**MATHEMATICS**

**SPECIALIST**

**3CD**

**Semester Two Examination 2010**

**MARKING KEY and SOLUTIONS**

**Section Two**

**Calculator-Assumed**

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**Question 8 [8 marks]**

Evaluate the following definite integrals **exactly** using Calculus and algebraic techniques :

a.  Put x = 2 sin u

[4 marks]

b. 

[4 marks]

**Question 9 [14 marks]**

Consider the following transformation matrices in the co-ordinate plane :

R rotate 90o clockwise about the origin

D dilate vertically about the origin with dilation factor 1.5

S horizontal shear parallel to the x axis with shear factor -1.

(a) Give matrices R, D and S.

[3 marks]

The 3 diagrams below show a rectangle.

Diagram 1 Diagram 2

Draw the image of this rectangle under the action of transformation :

(b) D (on Diagram 1)

[1 mark]

(c) S (on Diagram 2)

[2 marks]

(d) R then S (on Diagram 3)

[2 marks]

Diagram 3

**Question 9 [14 marks]**

(e) If the rectangle is transformed by matrix R then S, what matrix will return the resultant image back to the original rectangle ?

[3 marks]

A student observes a change in the area of the image when working with matrix D. The student writes a conjecture about the area of the resultant image, where n is the number of times the original rectangle has been dilated by matrix D.

(f) Suggest a conjecture for the Area(Image) when the original rectangle has been dilated n times by matrix D.

Also test your conjecture.

[3 marks]

**Question 10 [7 marks]**

Three rival supermarkets compete for customers in the suburb of Innaloo. Market research surveys show that given a customer shops at a supermarket one week, there is a chance that they will change and shop at another supermarket the next week. The transition matrix T shown below summarises this :

Current Supermarket

C

B

A

T = 

Supermarket for NEXT week

A

B

C

(a) Explain in words what element t23 of matrix T represents.

[1 mark]

(b) Given that a customer is shopping at supermarket B this week, give the probabilities that the customer will be shopping at each of A, B or C in 2 weeks time.

[2 marks]

**Question 10 [7 marks]**

(c) If there are currently 40% shopping at supermarket A, 30% at B and 30% at C, give the proportions shopping at each supermarket in 4 weeks time.

[2 marks]

(d) Determine the long run proportions of customers shopping at each of the supermarkets A, B and C.

[2 marks]

**Question 11 [11 marks]**

The end of a solid triangular prism is sawn off so that its top face ∆ DEF is NOT parallel to the bottom face ∆ ABC. Point O is the origin and **©**, **j** and **k** are unit vectors in the direction of the positive x, positive y and positive z directions respectively.

O

C (2,8,0)

B (6,6,0)

(4,0,0) A

(4,0,8) D

E (6,6,4)

F (2,8,12)

**z**

**y**

**x**

1. Determine the unit vector in the direction of vector .

[2 marks]

(b) Give the vector equation for the line containing points E and F.

[2 marks]

**Question 11 [11 marks]**

© Determine the intercept of the line containing points E and F with the xy plane.

[2 marks]

(d) Give the vector equation for plane DEF.

[2 marks]

(e) State the normal vector for plane DEF.

[1 mark]

**Question 11 [11 marks]**

(f) Give the angle, correct to the nearest degree, between plane DEF and the xy plane.

[2 marks]

**Question 12 [10 marks]**

The graph of y =  is shown below, where -2 ≤ x ≤ 2.

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B

A

1. Determine the co-ordinates for points A and B.

[2 marks]

(b) Find the equation of the tangent to the curve at point A.

[4 marks]

**Question 12 [10 marks]**

© Show, **using the anti-derivative of the sine function**, that the exact area of the region shaded is equal to  square units.

[4 marks]

**Question 13 [8 marks]**

A cyclist moves according to the velocity time graph shown below over a period of 60 seconds. Initially the cyclist is 150 metres to the left of a sign post.



(a) Give the acceleration of the cyclist at t = 5 seconds.

[1 mark]

(b) Determine the position of maximum displacement of the cyclist from the sign post over the 60 second period.

[3 marks]

**Question 13 [8 marks]**

(c) Determine when, correct to the nearest 0.1 seconds, the cyclist will be cycling past the signpost.

[4 marks]

**Question 14 [10 marks]**

(a) i. Using z = eiθ, show that 2i sin θ = eiθ - e-iθ

[1 mark]

ii. Hence prove that : 16 sin5θ = sin 5θ - 5 sin 3θ + 10 sin θ

[4 marks]

**Question 14 [10 marks]**

(b) i. Solve, in exact polar form, the complex equation z5 + 1 = 0.

[3 marks]

ii. Hence or otherwise prove that  .

[2 marks]

**Question 15 [12 marks]**

In this era of electronic communications, information is quickly shared. A rumour is started in a small country town with a population of 1 000. A small group of 5 people decide to start a rumour that “the town’s pub is about to permanently close”.

Let N(t) = the number of people who have heard the rumour after t hours

The rate of spread of the rumour is proportional to both the number of people who have already heard the rumour AND the number of people who have NOT heard the rumour. The rate of spread of the rumour is given by :

 0.0005 N(1000 - N)

(a) Using the above differential equation, explain what happens to the rate of spread of the rumour as the number of people who have heard the rumour rises.

[2 marks]

(b) Using the result  and the method of separation of variables, show that :

0.5t + k where k is some constant

[4 marks]

**Question 15 [12 marks]**

(c) Deduce the value of the constant k and hence determine the expression for N(t) in terms of t.

[4 marks]

It is considered that “everyone” in the town has heard the rumour when 99% of the town’s population have heard it.

(d) Determine how long (correct the nearest minute) it takes for “everyone” to have heard the rumour ?

[2 marks]